KINEMATIC SYNTHESIS OF PATH GENERATION OF PLANAR MECHANISMS

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Abstract: *Kinematic synthesis of planar mechanisms is studied. Path generation method is applied to generate planar four-bar mechanism. The task of synthesis refers to a problem in which a coupler point is desired to generate a given path. Nonlinear systems of equations with one constraint equations for each independent loop are obtained. Generated square equations with respect to the Lagrange coefficients are solved by using interpolation approximation method and design parameters are obtained. Results are given in tables.*

Keywords: planar mechanisms; path generation; kinematic synthesis.

1. INTRODUCTION

In the design process of planar and spatial mechanisms kinematic synthesis is the most common method for defining the construction parameters of the mechanism. After choosing the mechanism that will be used for a specific task, its construction parameters have to be designed with respect to the constraint conditions.

There are three main tasks in kinematic synthesis; function generation, motion generation, and path generation. Path generation is an example of constraint conditions which describes desired position and orientation for a certain point on the mechanism with respect to the fixed coordinate system. According to the given path of the point on the coupler, the construction parameters can be obtained by writing objective functions of the mechanism. Path generation and kinematic synthesis of mechanisms are a source of inspiration for many researchers. Input-output variables and design parameters were introduced in a polynomial equation by Levitskii [1]. Interpolation, Chebyshev and Least-Square approximations are used to calculate design parameters. Zimmerman [2] proposed a new algorithm for four precision points in the function generation of spherical four-bar mechanism. Polynomial approximation is used for three, four and five precision points in the study of Alizade [3], Alizade and Kilit [4], Murray and McCarthy [5] for the spherical four-bar mechanism. Davitashvili [6] presented synthesis and analysis of spherical mechanisms.

In this study, path generation kinematic synthesis is carried out for the planar four-bar mechanism to obtain the construction parameters of mechanism. In order to define the position of the four-bar mechanism with respect to the fixed frame, extra loop closure equations are formulated. Nonlinear system of equations are written for each independent loop. Precision points are determined by using Chebyshev spacing.

2. OBJECTIVE FUNCTION

Defining the objective function of the mechanism is the first task for kinematic synthesis of mechanisms. Objective function of the mechanism must be found and simplified with a suitable algebraic method. In this study, a planar four-bar mechanism that is shown in Figure 1 will be synthesized.

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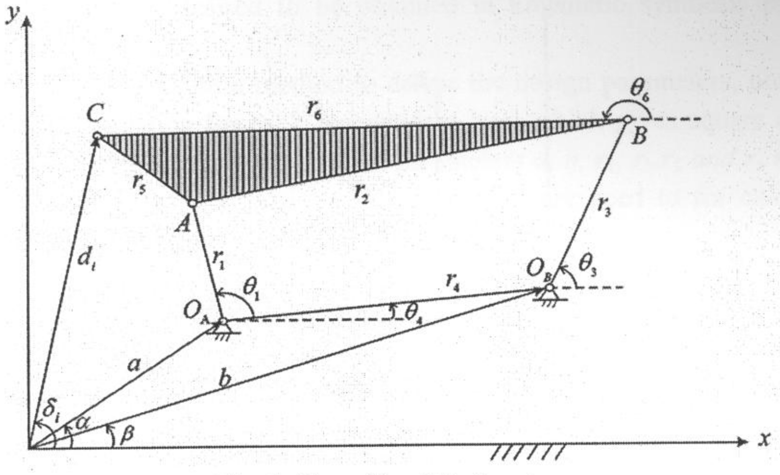

Fig. 1. Four-Bar Mechanism

According to unknown parameters of the mechanism, it is needed to write four loop closure equations to define all construction parameters:

$$\vec{a} = \vec{r_1} + \vec{r_5} + \vec{d}; \tag{1.a}$$

$$\vec{b} + \vec{r_3} + \vec{r_6} = \vec{d}; \tag{1.b}$$

$$\vec{r_2} = \vec{r_4} + \vec{r_3} - \vec{r_1}; \tag{1.c}$$

$$\vec{a} + \vec{r_4} = \vec{b}, \tag{1.d}$$

where the unknown parameters of the mechanism are; a, b, r_1, r_2, r_3, r_4, r_5 and r_6.

Equations (1.a, b, c, d) can be written in open form to obtain objective functions of the entire mechanism. Objective function which is obtained by using Equation (1.a) in the polynomial form is given in Equation (2):

$$P_1 f_{1i} + P_2 f_{2i} + P_3 = d_i^{\,2} + P_1 P_2 f_{3i}\,; \quad i = 1,2,3. \tag{2}$$

In Equation (2) $P_1 = a\,,\ \ P_2 = r_1,\ \ P_3 = r_5^{\,2} - a^2 - r_1^{\,2},\ \ f_{1i} = d_i \cos(\delta_i - \alpha)\,,\ \ f_{2i} = d_i \cos(\delta_i - \theta_{1i}),\ \ f_{3i} = 2\cos(\alpha - \theta_{1i})$ and θ_{1i} is the input angle of the four-bar mechanism.

Similarly, expanding and rearranging the loop closure equation of the mechanism given in Equation (1-b), second objective function is obtained as follows:

$$P_4 f_{4i} + P_5 f_{5i} + P_6 = d_i^{\,2} + P_4 P_5 f_{6i}\ ; \quad i = 1,2,3, \tag{3}$$

where

$P_4 = b\,,\ P_5 = r_3,\ P_6 = r_6^{\,2} + b^2 + r_3^{\,2},\ f_{4i} = 2d_i \cos(\delta_i - \beta), f_{5i} = 2d_i \cos(\delta_i - \theta_{3i}),$
$f_{6i} = 2\sin(\beta - \theta_{3i})$ and θ_{3i} is the output angle of the four-bar mechanism.

Thus, eight unknown construction parameters which belong to the mechanism, should be determined by solving the synthesis problem.

3. GIVEN PATH VARIABLES

The problem statement of the synthesis of four-bar mechanism is introduced by writing loop closure equations and obtaining objective functions of the mechanism. In this section, variables of the desired path, which is need to solve nonlinear equations that are given in the previous section, will be examined. Desired path of the mechanism is given for point C as shown in Figure 2. The function of the path is given as $y=x^2$ with respect to $x_i y_i$ coordinate frame.

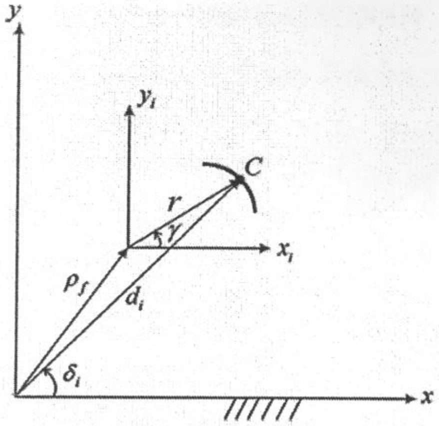

Fig. 2. Given Path of Point *C*

The position of point *C* with respect to fixed coordinate frame can be easily obtained by using the given position of the point *C* with respect to moving frame. The vector d_i and the angle δ_i which are used in Equations (1), (2) and (3), define the position of Point *C* on the mechanism with respect to the fixed coordinate frame.

The minimum and maximum values of the independent variable *x* which is defining the path of Point *C,* is called as x_0 and x_m, respectively. Precision points of the path are selected to be within x_0 and x_m and are found by Chebyshev spacing for three precision points:

$$x_i = \frac{x_0+x_m}{2} + \frac{(x_0-x_m)}{2}\cos\left(\frac{i\pi}{n}\right)\ ; \quad i = 1,2,3, \tag{4}$$

where *n* is the total number of precision points.

In this step of calculation, it is needed to obtain the input and output angles of the mechanism which correspond to three precision points. The minimum and maximum values of the input and output angles are, respectively: θ_{10} and θ_{1m}; θ_{30} and θ_{3m}.

4. OBTAINING UNKNOWN PARAMETERS

Equations (2) and (3) are the objective functions of the mechanism with the unknown coefficients P_1, P_2, P_3, P_4, P_5, P_6 and known functions f_{1i}, f_{2i}, f_{3i}, f_{4i}, f_{5i}, f_{6i} , where i=1, 2 and 3. Defining nonlinear parameters as λ_1 and λ_2 and rearranging the coefficients, P_i, in terms of linear and nonlinear terms Equations (5) and (6) are formulated:

$$\lambda_1 = P_1P_2\ ,\ P_k = l_k + \lambda_1 m_k\ ; \qquad k = 1,2,3; \tag{5}$$

$$\lambda_2 = P_4P_5\ ,\ P_k = l_k + \lambda_2 m_k\ ; \qquad k = 4,5,6. \tag{6}$$

Equation (2) is rewritten by separating linear and nonlinear terms and by using the relations given by Equation (5) as follows:

$$l_1 f_{1i} + l_2 f_{2i} + l_3 = {d_i}^2; \tag{7}$$

$$m_1 f_{1i} + m_2 f_{2i} + m_3 f_{3i} = f_{3i}. \tag{7.a}$$

Similarly, Equation (3) is rewritten by using the parameters given in Equation (6):

$$l_4 f_{4i} + l_5 f_{5i} + l_6 = {d_i}^2; \tag{8}$$

$$m_4 f_{4i} + m_5 f_{5i} + m_6 = f_{6i}. \tag{8.a}$$

The unknown coefficients l_i and m_i are solved from six equations that can be generated for three precision points making using of Equations (7), (7.a), (8), (8.a) given above. Thus, the unknown coefficients P_1, P_2, P_3, P_4, P_5, P_6, which are defined by Equations (2) and (3) can be found by using the relations given by Equations (5) and (6). These six coefficients are then used to calculate the construction parameters which are desired to be obtained in kinematic synthesis problem of the mechanism.

Using interpolation approximation method to define the design parameters, nonlinear system equation with one constraint equation for each independent loop which gives square equations with respect to the Lagrange coefficients is created. Design parameters *a, b,* r_1, r_3,r_5 *and* r_6 are found from two nonlinear loop equations. Finally, Equations (1.c) and (1.d) are used to for calculation of the remaining design parameters, r_2 and r_4.

5. NUMERICAL EXAMPLE

In the numerical example, the input angle interval is chosen as $\pi/6 < \theta_{1i} < \pi/2$ and the output angle interval is chosen as $\pi/4 < \theta_{3i} < \pi/3$. Desired path of point *C* is selected to be defined by the equation, $y = x^2$, within the interval $1 \le x \le 2$ and Chebyshev spacing is used to determine three precision points which are given in Table 1. Furthermore, the angles α and β which are shown in Fig.1 are chosen as $\alpha=\pi/3$ *rad* and $\beta=\pi/6$ *rad*. The vector d_i and the angle δ_i, which are used in Equations (1), (2) and (3), are path variables of point *C*. The path variables of *C* are calculated with respect to the selected parameters, r=1, the coordinate of the x_iy_i frame with respect to the fixed frame, ρ_f=(5, 10), and the angle γ which is chosen as $0 < \gamma < \pi/2$.

Table 1. Precision points for numerical example

Precision Points	*i*=1	*i*=2	*i*=3
θ_{1i} (rad)	0,78539	1.04719	1.308996
θ_{3i} (rad)	0,820305	0.894481	0.959931

The interpolation approximation method is applied to find the unknown construction parameters. The results for the calculated construction parameters are given in Table 2.

Table 2. Calculated construction parameters for numerical example

a	r_1	r_5	*b*	r_3	r_6	r_4	r_2
31.963	0.141	28.2855	33.8107	2.4405	22.1312	17.1168	19.936

6. CONCLUSION

Design parameters of a four-bar mechanism are obtained by using the given path variables of the coupler. Independent loop closure equations are written for each constraint of the mechanism to obtain the objective functions. These nonlinear systems of equations are solved for finding the construction parameters of the mechanism which satisfies the desired path.

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КИНЕМАТИЧЕСКИЙ СИНТЕЗ ПЛОСКИХ НАПРАВЛЯЮЩИХ МЕХАНИЗМОВ

Мехмет Исмет Джан Деде, Дуйгу Чомен, Илкай Эркылынчоглы, Эмре Узуноглы

Резюме: Рассмотрена проблема кинематического синтеза плоских механизмов. Метод применен для генерирования направляющих четырехзвенных и шестизвенных механизмов. Задача синтеза относится к проблеме генерирования заданной кривой посредством шатунной точки. Представлены нелинейные системы уравнений с одним уравнением связи для каждого независимого контура. Решение задачи сведена к решению квадратного уравнения относительно коэффициента Лагранжа при помощи метода приближения интерполяции и получены синтезируемые параметры механизма, которые приведены в таблицах.