KINEMATIC SYNTHESIS OF THE MOTION GENERATION OF LINKAGES

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Abstract: *The kinematic synthesis of two degree-of-freedom (DoF) manipulators is studied. Firstly, the motion generation requirements of manipulators are established according to the problem statement. Mathematical models for three and four poses of the gripper are determined. The motion generation synthesis of planar manipulators as a systematic methodology is presented. Dimensions of two-DoF manipulator are determined by interpolation approximation method. Finally combining the two real synthesis solutions of two-DoF manipulator kinematic synthesis for motion generation of four-bar linkage is accomplished.*

Keywords: motion generation; kinematic synthesis; planar four-bar linkage.

1. INTRODUCTION

Mechanism synthesis problem utilizes computational resources and tools by the definition of link lengths of given design requirements. Kinematic synthesis considers motion of links without taking into account the forces or moments. In general, mechanism synthesis problem can be divided into three tasks; synthesis of function generation, path generation and motion generation. [1-4]. These synthesis problems can be solved by graphical [5], analytical [6], or optimization [7] methods. The methodologies that are used for synthesis tasks can be categorized into three subgroups as; precision points, rigid body guidance, and approximate synthesis. If the number of synthesis parameters is equal to the number of precision points or rigid body guidance, usually interpolation approximation [8, 9], or best uniform approximation is used [10]. If the number of synthesis parameters is less than the number of synthesis equations, quadratic approximation and optimization methods are usually used. Numerical techniques are commonly combined with various optimization methods such as generic algorithms [11], evolutionary techniques [12], interior-point method [13], Gauss-constrained method [14] and Newton-Raphson method [15].

Most processes involve repetitive operation in which the same operation (position and orientation) is repeated continuously. A single-DoF linkage is sufficient for these tasks.

In this paper, firstly, the synthesis of two-DoF manipulator for certain operations, generally termed motion generation is studied. Moreover, kinematic synthesis presented is focused on the guidance of the gripper through three and then four positions (points). Interpolation approximation synthesis (exact synthesis) is used in solving motion generation problems. Secondly, combining two real solutions of synthesis of the two-DoF manipulators, one-DoF planar four-bar linkage which generates the motion of the coupler output link is obtained. This methodology is then examined through numerical examples for three- and four-position cases.

2. THREE POSITION SYNTHESIS OF TWO-DOF PLANAR MANIPULATOR

Planar two-DoF manipulator in Fig. 1 constructed with two pivot joints located at the origin of the coordinate systems $O_1x_1y_1$ and $O_2x_2y_2$ at points O_1 and O_2. The $O_0x_0y_0$ reference coordinate system is located so that the x_0-axis points to the right along the horizontal direction and its origin is located at the initial position of the moving gripper coordinate system $O_3x_3y_3$. Since the joint axes z_i are parallel to each other, all the twist angles α_i and the offset joint distance d_i are zero. The x_i axes are perpendicular to the joint axes z_i and the y_i axes are determined according to the right hand rule. Thus, there are four design parameters, a_0, θ_0, a_1 and a_2; and two joint variables, θ_1 and θ_2. The parameters ρ_x, ρ_y and θ_3 ($\theta_3 = \theta_2$) are position and orientation parameters of moving coordinate system $O_3x_3y_3$ with respect to the reference frame $O_0x_0y_0$.

In the case of three positions, allowing the arbitrary assignment of parameters a_2, a_1 and the location of the pivot O_1, reduces the problem to a simple geometric solution of finding the intersection of two segment medians. However, for the arbitrary assignment of parameter θ_0, the synthesis problem leads to a system of three nonlinear algebraic equations with a_0, a_1 and a_2 as unknowns. Finite solutions are available by using superposition method.

Vector equation of the system is written in Eq. (1) by replacing the links of the two-DoF manipulator with the vectors presented in Fig. 1

$$\vec{a}_1 = \vec{\rho} - \vec{a}_0 - \vec{a}_2. \tag{1}$$

In complex polar notation, Eq. (1) can be written as:

$$a_1 e^{j\theta_1} = \rho e^{j\alpha} - a_0 e^{j\theta_0} - a_2 e^{j\theta_2}. \tag{2}$$

If Eq. (2) is transformed into complex rectangular form, two algebraic equations are obtained by separating real and imaginary parts of the equation:

$$\begin{aligned} a_1 \cos\theta_1 &= \rho_x - a_0 \cos\theta_0 - a_2 \cos\theta_2; \\ a_1 \sin\theta_1 &= \rho_y - a_0 \sin\theta_0 - a_2 \sin\theta_2. \end{aligned} \tag{3}$$

Both sides of Eq. (3) are squared and added together to eliminate the angle θ_1. Thus, the general equation for synthesis problem is obtained as shown in Eq. (4)

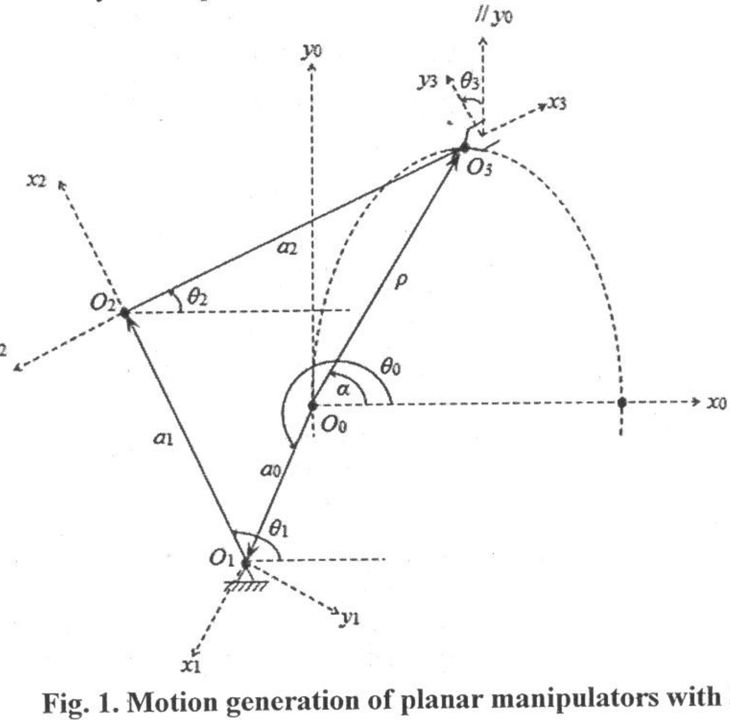

Fig. 1. Motion generation of planar manipulators with 3 positions

$$\begin{aligned} a_1^2 - a_2^2 - a_0^2 + 2a_0(\rho_x \cos\theta_0 + \rho_y \sin\theta_0) + 2a_2(\rho_x \cos\theta_2 + \rho_y \sin\theta_2) \\ - \rho^2 - 2a_0 a_2 \cos(\theta_2 - \theta_0) = 0. \end{aligned} \tag{4}$$

Every term in Eq. (4) is divided by the factor a_0a_2 and rearranged for the synthesis of three precision positions in Eq. (5)

$$(a_1^{\,2}-a_2^{\,2}-a_0^{\,2})(a_0\,a_2)^{-1}+2a_2^{\,-1}(\rho_{xi}\cos\theta_0+\rho_{yi}\sin\theta_0)+2a_0^{\,-1}(\rho_{xi}\cos\theta_2+\rho_{yi}\sin\theta_2)$$
$$+(a_0\,a_2)^{-1}(-\rho_i^{\,2})-2\cos(\theta_{2i}-\theta_0)=0, \qquad i=1,2,3. \tag{5}$$

In this case, for three positions of gripper, three unknowns a_0, a_1, a_2 are to be solved. If parameter θ_0 is given, the synthesis problem can be formulated to the solution of these three unknowns by using data set of position and orientation of moving coordinate system $O_3x_3y_3$, i=1, 2, 3, which is attached to the gripper.

Synthesis Eq. (5) can be introduced in polynomial form as:

$$P_1\,f_{1i}+P_2\,f_{2i}+P_3\,f_{3i}+P_4\,f_{4i}-F_i=0, \qquad i=1,2,3, \tag{6}$$

where

$P_1=\left(a_1^2-a_2^2-a_0^2\right)\left(a_0\,a_2\right)^{-1}$; $f_{1i}=1$; $P_2=a_2^{-1}$; $f_{2i}=2\left(\rho_{xi}\cos\theta_0+\rho_{yi}\sin\theta_0\right)$; $P_3=a_0^{-1}$; $f_{3i}=2\left(\rho_{xi}\cos\theta_{2i}+\rho_{yi}\sin\theta_{2i}\right)$; $P_4=\left(a_0\,a_2\right)^{-1}$; $f_{4i}=-\rho_i^2=-\left(\rho_{xi}^2+\rho_{yi}^2\right)$; $F_i=2\cos\left(\theta_{2i}-\theta_0\right)$.

Eq. (6) describes four unknowns however, there are only three linear equations. If the number of unknown is more than the number of linear equations, this will result in undetermined linear system of equations. Thus, equation is nonlinear and one more equation is required. Additional equation can be introduced between constant parameters as $P_4= P_2\,P_3$.

If $P_4=\lambda$ is defined as non-linear parameters, there will be four equations and four unknowns as follows:

$$P_1f_{1i}+P_2f_{2i}+P_3f_{3i}=F_i-\lambda\,f_{4i}, \qquad i=1,2,3; \tag{7}$$

$$P_2P_3-\lambda=0. \tag{8}$$

Since the constant parameters P_i on the left hand side of Eq. (7) are linear, it means that the synthesis parameters are linearly proportional with non-linear parameters λ as:

$$P_k=l_k+\lambda\,m_k, \qquad k=1,2,3, \tag{9}$$

where l_k and m_k are real and non-linear parts respectively.

Substituting Eq. (9) into Eq. (7) and equating the real and non-linear terms separately, two real system equations with respect to the unknown parameters l_k and m_k are obtained as:

$$l_1\,f_{1i}+l_2\,f_{2i}+l_3\,f_{3i}=F_i; \tag{10}$$

$$m_1\,f_{1i}+m_2\,f_{2i}+m_3\,f_{3i}=-f_{4i}, \qquad i=1,2,3. \tag{11}$$

Using interpolation approximation method, the parameters l_k and m_k, k=1, 2, 3 are computed by Cramer's rule from linear Equations (10) and (11). Matrix $[f_{ki}]$, k, i=1, 2, 3 must be non-singular to find a solution. It is noted that, a square matrix is non-singular if and only if its determinant is non-zero. Now substituting Eq. (9) for k=2, 3 into Eq. (8) a second order equation is derived as shown in Eq. (12).

$$p\,\lambda^2+q\,\lambda+r=0, \tag{12}$$

where

$$p=m_2\,m_3;\; q=l_2\,m_3+l_3\,m_2;\; r=l_2\,l_3.$$

The non-linear parameter λ in Eq. (12) and parameters l_k and m_k in Eq. (10) and (11) are used to find P_k, k=1, 2, 3 in Eq. (9). Finally, using these P_k values, all synthesis parameters of the two-DoF manipulator, a_0, a_1, a_2, can be found from Eq. (6) as:

$$a_0=P_3^{-1}, \quad a_1=(P_1P_2^{-1}P_3^{-1}+P_2^{-2}+P_3^{-2})^{0.5}, \quad a_2=P_2^{-1}. \tag{13}$$

2.1. Numerical Example for Three Precision Positions Case

The three positions of moving coordinate system $O_3x_3y_3$ as the three precision positions of the gripper in Fig. 2 are given by ρ_x= {0; 27.375; 32.5}, ρ_y={0; 15.35; 0}, θ_2={14°; 4°; -28°} and θ_0=280°. Using given data and Eq. (6), the elements of matrix $[f_{ki}]$, k, i=1, 2, 3 and column vectors $[F]$ and $[f_4]$ are calculated. The linear and non-linear components of constant parameters as; l={-0.139513; 0.03158; 0.01767} and m={0; 1.86833; 18.0369} are computed to solve the linear Eq. (10) and (11). Using the magnitudes of l_k and m_k Eq. (12) is obtained as 33.6987 λ^2-0.39732 λ+0.000558=0 and two real solutions of this square equation are found as λ= {0.00163032; 0,0101603}. Two solutions for polynomial constant parameters of Eq. (9) are obtained as given below with respect to the two values of non-linear parameters and the magnitudes of parameters l_k and m_k:

Solution 1: P_1=-0.139513; P_2=0.0346286; P_3=0.04708.

Solution 2: P_1=-0.139513; P_2=0.0505654; P_3=0.200934.

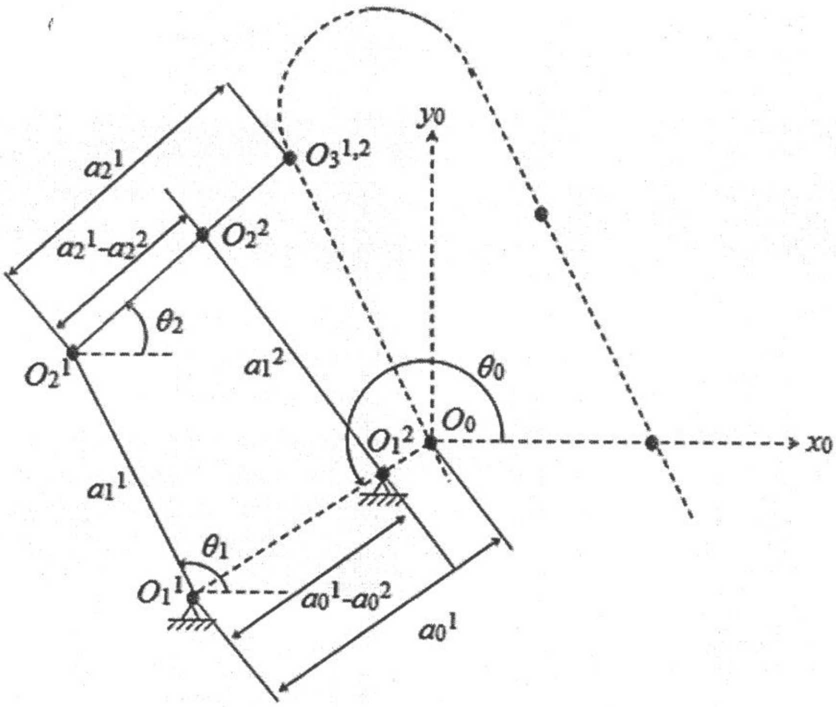

Fig. 2. Motion generation of four-bar linkage

Finally, design parameters of two manipulators with the same motion generation of the gripper are calculated by using Eq. (13).

Manipulator 1: a_0=21.2404; θ_0=280°; a_1=34.634; a_2=28.8779.

Manipulator 2: a_0=4.97676; θ_0=280°; a_1=20.0535; a_2=19.7764.

In Fig. 2, the two real solutions for the design parameters of a two-DoF serial manipulator are combined to form a single four-bar linkage. It can be concluded from this methodology that "In order to design motion generation of four-bar linkage it is necessary and sufficient to combine the two real solutions accomplished for motion generation of two-DoF manipulators" As it is observed in Fig. 2, the design parameters of new four-bar linkage for gripper guidance are determined as: fixed link $a_0'=(a_0)_1-(a_0)_2$=18.1618; orientation of fixed link θ_0=280°; coupler link $a_2'=(a_2)_1-(a_2)_2$=10.1751; follower links $(a_1)_1$=34.634, $(a_1)_2$=20.0535.

3. FOUR POSISIONS SYNTHESIS OF TWO-DOF PLANAR MANIPULATOR

In this subsection, the main goal of motion generation synthesis is to design a two-DoF planar manipulator to pass through four positions. Following set of equations can be formulated by using Eq. (4) for four positions synthesis case.

$$\sum_{k=1}^{6} P_k f_{ki} = F_i \quad i = 1,...,4, \tag{14}$$

where

$P_1 = a_1^2 - a_2^2 - a_0^2$; $P_2 = a_0 \cos\theta_0$; $P_3 = a_0 \sin\theta_0$; $P_4 = a_2$; $P_5 = a_0\, a_2 \cos\theta_0$; $P_6 = a_0\, a_2 \sin\theta_0$;

$f_{1i} = 1$, $f_{2i} = 2\rho_{xi}$, $f_{3i} = 2\rho_{yi}$, $f_{4i} = 2\rho_{xi}\cos\theta_{2i} + 2\rho_{yi}\sin\theta_{2i}$, $f_{5i} = -2\cos\theta_{2i}$, $f_{6i} = -2\sin\theta_{2i}$

$F_i = (\rho_{xi}^2 + \rho_{yi}^2)$.

In this problem, there are six unknowns and four linear equations. Thus, Eq. (14) presents an undetermined linear system of equations. Two additional equations between constant parameters are formulated to find an analytical solution of non-linear Eq. (14):

$$P_5 = P_2 P_4 \text{ and } P_6 = P_3 P_4. \tag{15}$$

If the non-linear parameters are designated by $P_5 = \lambda_1$ and $P_6 = \lambda_2$ respectively, Eq. (14) and (15) will be modified as shown in Eq. (16)

$$\sum_{k=1}^{4} P_k f_{ki} = F_i - \lambda_1 f_{5i} - \lambda_2 f_{6i},\quad P_2 P_4 - \lambda_1 = 0,\quad P_3 P_4 - \lambda_2 = 0,\quad i = 1,...,4. \tag{16}$$

Therefore, "six equations with six unknowns" requirement is achieved. The left hand side of the first system of Eq. (16) is linear. The constant parameters are selected to be linearly proportional with λ_1 and λ_2 as:

$$P_k = l_k + \lambda_1 m_k + \lambda_2 n_k,\quad k = 1,...,4. \tag{17}$$

Substituting Eq. (17) into the first equation of Eq. (16) and equating the real and non-real terms separately, three real system equations with respect to the unknown parameters l_k, m_k, n_k are determined:

$$\sum_{k=1}^{4} l_k f_{ki} = F_i,\quad \sum_{k=1}^{4} m_k f_{ki} = -f_{5i},\quad \sum_{k=1}^{4} n_k f_{ki} = -f_{6i},\quad i = 1,...,4. \tag{18}$$

$[f_{ki}]$ matrix must be non-linear to find a solution. If determinant of the matrix is not equal to zero ($|f_{ki}| \neq 0$), the solution of system of equations in Eq. (18) gives l_k, m_k, and n_k, k=1, ..,4.

Substitution of Eq. (17) into second and third equations of Eq. (16) results:

$$(l_2 + \lambda_1 m_2 + \lambda_2 n_2)(l_4 + \lambda_1 m_4 + \lambda_2 n_4) - \lambda_1 = 0; \tag{19}$$

$$(l_3 + \lambda_1 m_3 + \lambda_2 n_3)(l_4 + \lambda_1 m_4 + \lambda_2 n_4) - \lambda_2 = 0. \tag{20}$$

The open form of Eq. (19) and (20) are presented in Eq. (21) and (22).

$$a_1\lambda_1^2 + a_2\lambda_2^2 + a_3\lambda_1\lambda_2 + a_4\lambda_1 + a_5\lambda_2 + a_6 = 0; \tag{21}$$

$$b_1\lambda_1^2 + b_2\lambda_2^2 + b_3\lambda_1\lambda_2 + b_4\lambda_1 + b_5\lambda_2 + b_6 = 0, \tag{22}$$

where

$a_1 = m_2 m_4$; $a_2 = n_2 n_4$; $a_3 = m_2 n_4 + m_4 n_2$; $a_4 = l_2 m_4 + l_4 m_2 - 1$; $a_5 = l_2 n_4 + l_4 n_2$; $a_6 = l_2 l_4$;

$b_1 = m_3 m_4$; $b_2 = n_3 n_4$; $b_3 = m_3 n_4 + m_4 n_3$; $b_4 = l_3 m_4 + l_4 m_3 - 1$; $b_5 = l_3 n_4 + l_4 n_3 + 1$; $b_6 = l_3 l_4$.

Multiplying Eq. (22) with $-a_2 b_2^{-1}$ and after summation with the Eq. (21):

$$\lambda_2 = (c_1\lambda_1^2 + c_2\lambda_1 + c_3)(c_4\lambda_1 + c_5)^{-1}, \tag{23}$$

where

$$c_1 = \begin{vmatrix} a_2 & b_2 \\ a_1 & b_1 \end{vmatrix}, \; c_2 = \begin{vmatrix} a_2 & b_2 \\ a_4 & b_4 \end{vmatrix}, \; c_3 = \begin{vmatrix} a_2 & b_2 \\ a_6 & b_6 \end{vmatrix}, \; c_4 = \begin{vmatrix} a_3 & b_3 \\ a_2 & b_2 \end{vmatrix}, \; c_5 = \begin{vmatrix} a_5 & b_5 \\ a_2 & b_2 \end{vmatrix}.$$

Substituting λ_2 and λ_2^2 in Eq. (23), a polynomial equation of fourth-order is obtained as shown in Eq. (24)

$$\sum_{i=1}^{4} d_i \lambda_1^i + d_0 = 0, \tag{24}$$

where $d_4 = a_1 c_4^2 + a_2 c_1^2 + a_3 c_1 c_4$.

It is important to see that the magnitude of d_4 is zero, if the values of a_1, a_2, a_3, c_1 and c_4 are used from Eq. (23). Then, the polynomial in Eq. (24) is reduced to the third order:

$$\sum_{i=1}^{3} d_i \lambda_1^i + d_0 = 0, \tag{25}$$

where

$$d_0 = a_2 c_3^2 + a_6 c_5^2 + a_5 c_3 c_5; \; d_1 = 2\, a_2 c_2 c_3 + a_3 c_3 c_5 + a_4 c_5^2 + 2\, a_6 c_4 c_5 + a_5 c_2 c_5 + a_5 c_3 c_4;$$

$$d_2 = a_1 c_5^2 + a_2 c_2^2 + 2a_2 c_1 c_3 + a_3 c_2 c_5 + a_3 c_3 c_4 + 2a_4 c_4 c_5 + a_6 c_4^2 + a_5 c_1 c_5 + a_5 c_2 c_4;$$

$$d_3 = 2a_1 c_4 c_5 + 2a_2 c_1 c_2 + a_3 c_1 c_5 + a_3 c_2 c_4 + a_4 c_4^2 + a_5 c_1 c_4.$$

Analytical solution of third order polynomial Eq. (25) can be calculated by using symbolic mathematical calculation software. In this study the Mathematica software is utilized for this purpose.

3.1. Numerical Example 1

Given four positions of the gripper position and orientation in Fig. 3 are ρ_x={0; 25; 18.75; 35}, ρ_y={0; 25; 18.75; 0}, θ_2={14°; 10°; -4°; -28°}.

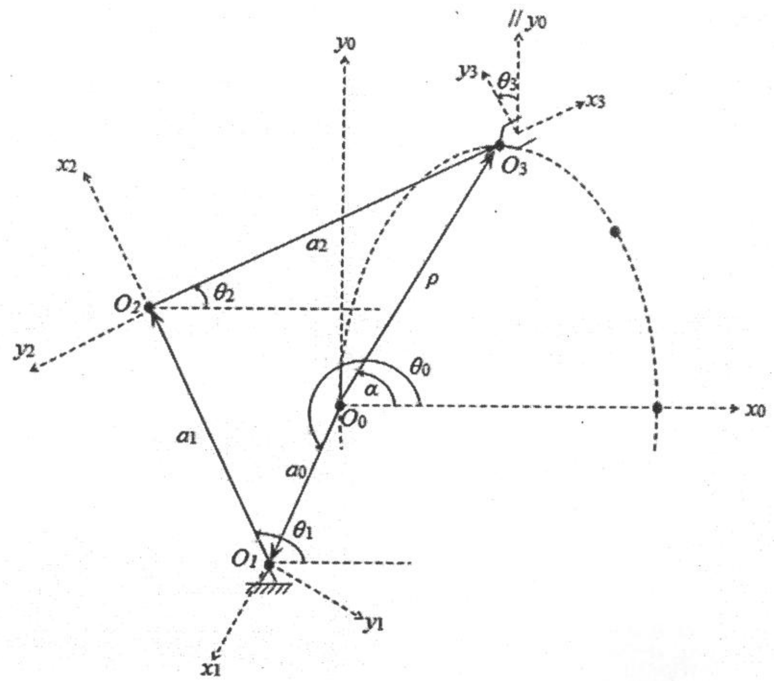

Fig. 3. Motion generation of planar manipulators with 4 positions

By using given data and Eq. (14), the elements of matrix $[f_{ki}]$ and column vectors $[F]$, $[f_5]$ and $[f_6]$, (k & i = 1, ..., 4) are computed. Using system of Eq. (14), the linear components of constant parameters are calculated as, l={0; -6.425; 0.0344; 27.0975}, m={1.94059; 0.00085; 0.00411; -0.0037}, n={0.4838; -0.7350; 0.0010; 0.06022}. Using values of l_k, m_k, and n_k, the Eq. (25) will be reduced to $2.93\ 10^{-10}\ \lambda_1^3 - 2.42\ 10^{-6}\ \lambda_1^2 + 0.049\ \lambda_1 + 0.76065 = 0$. Solution of the third order equation gives two imaginary roots and one real root as λ_1= -144.68. The value of the second non-linear parameter can be computed from Eq. (23) as λ_2= -15.4183. With respect to the Eq. (18), constant parameters are obtained as P_1= -288.225, P_2= -5.41527, P_3= -0.577096 and P_4= 26.717. The errors in the precision positions are calculated by using Eq. (14) as Δ_1= -1.20792.10^{-13}, Δ_2= 0, Δ_3= -1.13687.10^{-13} and Δ_4= -2.27374.10^{-13}. Finally, the design parameters of two-DoF manipulator with motion generation of gripper are obtained as follows:

$$a_2 = P_4 = 26.717,\ \theta_0 = \tan^{-1}\frac{P_3}{P_2} = 186.03^\circ,\ a_0 = \frac{P_2}{\cos\theta_0} = 5.4459,\ a_1 = \sqrt{P_1 + a_2^2 + a_0^2} = 21.3362.$$

3.2. Numerical Example 2

In the first solution just one real solution is obtained. To complete motion generation of four-bar mechanism, the position and orientation of gripper are changed to the given workspace as ρ_x={0, 9.897, 27.9903, 30}, ρ_y={0, 16.3453, 9, 0} and θ_2={15°, 5°,-5°,-15°}. First, the elements of matrix $[f_{ki}]$, and the column vectors $[F]$, $[f_5]$ and $[f_6]$ (k & i = 1, ..., 4) are computed. Following this computation, from Eq. (18), constant parameters: l={0, 7.93454, 1.3234, 7.3147}, m={1.9318, 0.0148, 0.0034, -0.0153}, n={0.5176, 0.1329, 0.0163, -0.1554} are calculated. Numerical value of polynomial equation in Eq. (25) is $1.67192.10^{-9}\ \lambda_1^3 + 0.000035\ \lambda_1^2 + 0.023895\ \lambda_1 - 1.31586 = 0$. Third order equation has three real roots. Then, the second non-linear parameter is calculated by using Eq. (23). Constant parameters in Eq. (17) are computed for each numerical value of λ_1 and λ_2. All numerical values are listed in Table 1.

Table 1. Numerical values of parameters

Real Roots	λ_1	λ_2	P_1	P_2	P_3	P_4	a_0	θ_0	a_1	a_2
1	-740.319	-123.914	-1494.33	-19.5116	-3.265	37.9425	19.783	9.4996°	18.3485	37.9425
2	51.1294	8.5356	103.193	9.8271	1.6405	5.2029	9.9631	9.4776°	15.1501	5.2029
3	-20792.4	3186.69	-38518.3	123.129	-18.8709	-168.868	124.566	-8.71347°	74.2628	-168.868

It is clear that the third solution is not physically realizable due to minus sign of a_2 value. Therefore, a four-bar mechanism generating precision positions can be constructed by using the first and the second solutions.

4. CONCLUSIONS

In this paper, the motion generation problem of two- DoF manipulator is solved for three and four precision positions of grippers. Combining the two real solutions of synthesis motion generation manipulator, four-bar mechanism that can generate the specified positions was able to be created. The theory is complemented with numerical solutions for both three-positions and four-positions cases.

5. ACKNOWLEDGMENTS

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КИНЕМАТИЧЕСКИЙ СИНТЕЗ МЕХАНИЗМОВ ПО ЗАДАННОМУ ДВИЖЕНИЮ ТВЕРДОГО ТЕЛА

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Резюме: Представлена задача кинематического синтеза двухстепенного манипулятора. Вначале рассматривается задача по воспроизведению движения согласно проблеме. Определяются математические модели по воспроизведению движения трех и четырех положений рабочего органа. Конструктивные параметры двухстепенного манипулятора определяются применением теории приближения функции, интерполированием. В заключении, объединяя два положительных рещения синтеза двухстепенного манипулятора получаем четырехзвенный механизм по воспроизведению положения и ориентации шатуна.