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CONTENTS

KINEMATIC SYNHTHESIS OF FUNCTION GENERATING SPATIAL RTSR MECHANISM BY CHEBYSHEV APPROXIMATION

Duygu Çömen*, İlkay Erkılınçoğlu, Mehmet İsmet Can Dede*****

(*Ph.D. Student, **M.Sc. Student, ***Assistant Professor, Izmir Institute of Technology, 35437, Izmir, Turkey)

Abstract: *In this study, RTSR mechanism is chosen to generate an output function by using the technique of Chebyshev approximation. Equal spacing helps to determine the precision points in the mechanism. The error value is added to the objective function and considered as a parameter during the calculation. The new precision points are obtained from the roots by taking the derivative of the objective function. The recursive process is carried out until the absolute error values are stabilized within the preset tolerances. Results are given in tables and graphs.*

Keywords: spatial RTSR mechanism; Chebyshev Approximation; precision points; kinematic synthesis; function generation

1. INTRODUCTION

Kinematic synthesis problem has an important role in the design process of the mechanisms. After selecting a mechanism that is kinematic synthesis will be analyzed for a certain task, its construction parameters must be determined according to the constraint conditions.

Function generation is one of the examples of the constraint conditions which describe a relation between motion output and motion input with a desired function. These mechanisms are a source of inspiration for many authors. Input-output variables and design parameters were introduced in a polynomial equation by Levitskii [1]. He calculated the design parameters by using Interpolation, Chebyshev and Least-Square approximation. Freudenstein [2] had an analytical approach to the design of planar four-bar linkage. Denavit and Hartenberg [3] presented the synthesis procedure for three precision points. Davitashvili [4] introduced dynamics of spherical mechanisms. Zimmerman [5] proposed a new algorithm for four precision points in the function generation of spherical four-bar mechanism. Polynomial approximation is used for three, four and five precision points in the study of Alizade [6], Alizade and Kilit [7], Murray and McCarthy [8] for the spherical four-bar mechanism. Davitashvili [9] presented synthesis and analysis of spherical mechanisms. Alizade and Kilit also present a new methodology, minimum deviation area (MDA), which is a method for selecting the precision points on given function through graphs. Sancibrian et al. [10] and Cervantes-Sanchez et al. [11] studied on function generation of spatial mechanisms. Sancibrian et al. [10] proposed a new synthesis method which uses a dimensional synthesis technique and local optimization. Cervantes-Sanchez et al. [11] had a new approach for three and four precision points for exact synthesis. Alizade and Gezgin [12] proposed a new function generation synthesis method for spherical four-bar mechanism with six independent parameters in which interpolation, least-square and Chebyshev approximation are used and error differences are compared with each other through graphs. Finally, Davitashvili [13] presented a research of spatial hinged mechanisms.

In this study, Chebyshev Approximation is used for the function generation of the spatial RTSR mechanism. Absolute error values are added to the objective function modified with a changing sign for each value. Precision points are determined using equal spacing and objective function is solved for the construction parameters and the error value. After determining the construction parameters, derivation of the objective function is used to obtain new precision points which are used for recalculation and reducing the error.

**Corresponding Author. Tel.: +90 232 750 6757. E-mail: duygucomen@iyte edu.tr*

2. RTSR MECHANISM

The RTSR mechanism which is shown in Figure 1, is composed of four joints and four links. The first link, r, is between the joints A and B which are revolute and Hooke's joints. Those joints' axes intersect at point A. The second link, c, is between the joints C and D which are constructed as a spherical and a revolute joint, respectively. These two free-end links are connected to each other with a third link, l, which is between the joints B and C. The third link, l, can rotate freely in three perpendicular directions as a result of these joint connections. The fourth link DA is connected to link r and link c by two separate revolute joints rotating about Y-axis and Z-axis respectively. The link c moves on XY plane and the link r moves on X'Z' plane which is parallel to XZ plane.

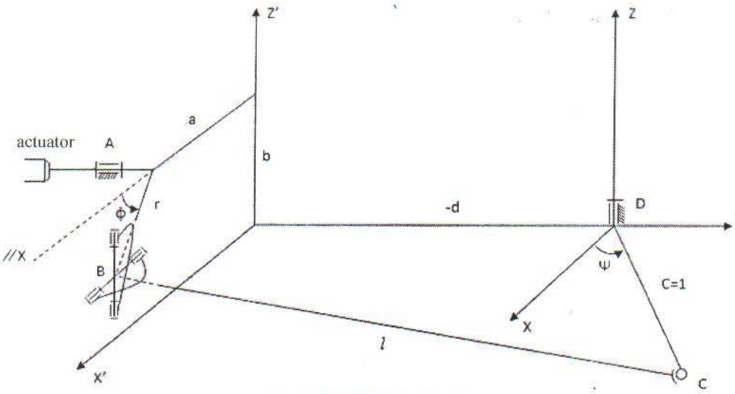

Fig. 1. RTSR Mechanism

3. OBJECTIVE FUNCTION

One of the main problems that may arise during the synthesis of any mechanism is describing the objective function of that mechanism. During this process, one has to take into account the constrains. Objective function of the mechanism must be found and simplified with a suitable algebraic method. One of the axes of spherical and Hooke's joints commonly intersect in one point. That's why algebraic method is a proper way for synthesis of the mechanisms.

In this study, the RTSR mechanism will be examined. In the task of writing the objective function of the mechanism, it is required to write the position vectors of points B and C with respect to the original coordinate frame XYZ:

$$B=[a+r\,Cos\phi \quad -d \quad b-r\,Sin\phi]^T; \qquad C=[c\,Cos\psi \quad c\,Sin\psi \quad 0]^T \tag{1}$$

The parameter c is considered as a known value, which is taken as $c=1$, to reduce the number of unknown parameters of the mechanism. Thus, 5 unknown construction parameters (r, l, a, b, d) which belong to the mechanism should be determined in our synthesis problem. A relation that gives the length l is written by using the positions of points B and C:

$$l^2=(X_C-X_B)^2+(Y_C-Y_B)^2+(Z_C-Z_B)^2. \tag{2}$$

Using Equation (1) and expanding the Equation (2), the objective function of the RTSR mechanism in the polynomial form is obtained as below:

$$P_0\,f_0(\phi_i)+P_1\,f_1(\phi_i)+P_2\,f_2(\phi_i)+P_3\,f_3(\phi_i)+P_4\,f_4(\phi_i)=F(\phi_i), \quad i=1..n+1, \tag{3}$$

where

$$P_0 = (l^2 - 1 - a^2 - b^2 - d^2 - r^2)/2r \,,\quad P_1 = a/r \,,\quad P_2 = -a \,,\quad P_3 = d/r \,,\quad P_4 = b$$

$$f_0 = 1 \,,\quad f_1 = Cos\psi \,,\quad f_2 = Cos\phi \,,\quad f_3 = Sin\psi \,,\quad f_4 = Sin\phi \,,\quad F(\phi) = -Cos\psi\, Cos\phi$$

c=1 and ϕ is the input angle and ψ is the output angle of the mechanism.

4. CHEBYSHEV APPROXIMATION

Chebyshev Approximation is a method that gives an approximate function with equal errors in the design interval. The function generated with Chebyshev Approximation Method oscillates between an error bound of $\pm L$ as it is shown in Figure 2.

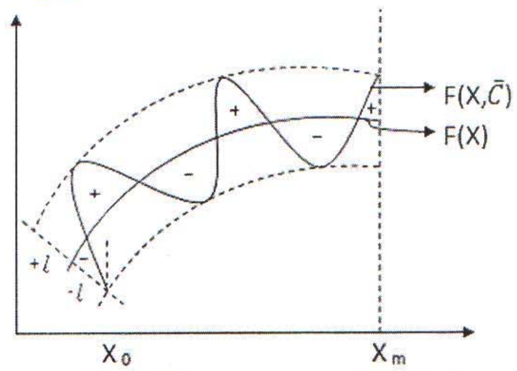

Fig. 2. Desired $(F(X))$ and Generated $(F(X,\overline{C}))$ Functions

The maximum absolute value of the difference between the desired function and generated function becomes L which is given below:

$$\left| F(\phi_i, \overline{C}) - F(\phi_i) \right| = L \,, \qquad i = 1 \ldots n+2 \,. \qquad (4)$$

The sign of the maximum error value of generated function changes in each section of the solution interval. Each section is denoted with a sign in Figure 2. It is noted that the boundaries of the sections correspond to precision points, which intersect with the desired function. The last form of the objective function is obtained by adding the error term with the changing sign to the Equation (4) and it is given in Equation (5):

$$\sum_{k=0}^{4} P_k\, f_k(\phi_i) = F(\phi_i) + (-1)^i\, L, \qquad i = 1 \ldots n+2. \qquad (5)$$

The absolute error value L is considered as an unknown parameter and solved with other construction parameters of the system. Thus the number of the equations becomes $n+2$, where $n+2$ is the number of the construction parameters which are to be found during the design process. In the case that is considered in this study, the total number of construction parameters is 6.

Equation (5) can be solved numerically for finding the coefficients P_0, P_1, P_2, P_3, P_4 and L, since there are six equations with six unknown parameters. By using these coefficients, construction parameters can be solved using the relations given in Equation (3).

It is needed to plot the derivative of the objective function to reduce the error value, where the roots of this derivation give the extremums of the objective function. These points will become the new precision points and the error value is reduced. There will be n+1 roots of the derivation, which are composed of n+1 precision points and the last required point is taken from the previous solution's initial or last boundary precision points. Derivative of the objective function in Equation (5) is given by the Equation (6):

$$\sum_{k=0}^{4} P_k f_k'(\phi_i) = F'(\phi_i), \qquad i = 1...n+2. \tag{6}$$

5. NUMERICAL EXAMPLE

As a numerical example, the input angle interval is chosen as 0.4 π < ϕ < π with the output angle interval 0.1 π < ψ < 0.6 π. Output function is chosen as $y = x^{0.8}$ with the interval $1 \le x \le 3$ and equal spacing is used to determine six precision points which are given in the first row of the Table1. Using these initial precision points, construction parameters and error values are calculated as shown in Table 1, 2. Then the roots of the Equation (6) are taken as new precision points. First boundary precision point from the previous solution is taken, while five points are taken from the roots of the derivation, since six precision points are required. This process is done recursively for 5 times and calculated construction parameters of RTSR mechanism are given in Table 2.

Table 1

Precision points for numerical example

Step	*Precision Points*	1	2	3	4	5	6
1	ϕ (rad)	1.52592	1.7952	2.06448	2.33375	2.60303	2.87231
	ψ (rad)	0.562555	0.800055	1.02902	1.25102	1.46716	1.67826
2	ϕ (rad)	1.52592	1.60967	1.89106	2.17389	2.47009	2.77421
	ψ (rad)	0.562555	0.637455	0.882453	1.11999	1.36113	1.6019
3	ϕ (rad)	1.52592	1.61013	1.84742	2.17745	2.51586	2.7705
	ψ (rad)	0.562555	0.637864	0.845071	1.12294	1.39778	1.59899
4	ϕ (rad)	1.52592	1.61172	1.84838	2.17746	2.51487	2.7687
	ψ (rad)	0.562555	0.639279	0.845896	1.12294	1.39699	1.59758
5	ϕ (rad)	1.52592	1.61196	1.84941	2.18072	2.51378	2.76508
	ψ (rad)	0.562555	0.639491	0.846783	1.12564	1.39612	1.59476

Table 2

Calculated construction parameters for numerical example

	a	b	d	*L*	r	L
1	-1.17153	6.42688	0.365598	6.64007	0.162715	6.08041×10^{-6}
2	-1.16958	6.40791	0.363991	6.62113	0.162967	3.75825×10^{-5}
3	-1.16983	6.40985	0.364169	6.62307	0.162947	3.89027×10^{-5}
4	-1.16983	6.40984	0.364169	6.62307	0.162947	3.89092×10^{-5}
5	-1.16982	6.40982	0.364166	6.62305	0.162947	3.89037×10^{-5}

It can be observed that calculated parameter values do not change significantly after 5 repetitions of the calculation. This result can be seen graphically by plotting together the error functions that belong to each calculation Fig. 3.

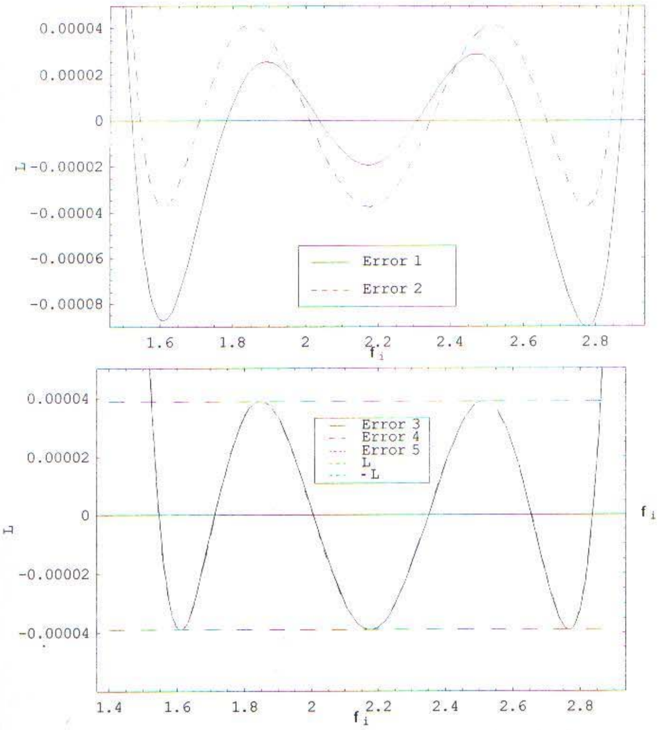

Fig. 3. Errors in objective function of the designed mechanism
a) first two calculations b) 3rd, 4th and 5th calculations

6. CONCLUSION

Chebyshev Approximation is applied for spatial RTSR mechanism. Precision points are determined by using the equal spacing and objective function is solved by numerical techniques. The solution is repeated for the new precision points and the error values are reduced by using the derivation of the objective function. It is observed that in the 5th repetition of the calculations, the objective function stays the same with the previous calculation step. The absolute error values similarly do not change significantly for the 4th and 5th calculations. Thus, it was possible to synthesis a RTSR mechanism for function generation within an error bound using Chebyshev Approximation.

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КИНЕМАТИЧЕСКИЙ СИНТЕЗ ПЕРЕДАТОЧНОГО ПРОСТРАНСТВЕННОГО МЕХАНИЗМА RTSR С ИСПОЛЬЗОВАНИЕМ ЧЕБЫШЕВСКОГО ПРИБЛИЖЕНИЯ

Дуйгу Чомен, Илкай Эркылынчоглу, М.И. Джан Деде

Резюме: В этой работе механизм RTSR представлен для генерирования выходной функции с использованием техники Чебышевского приближения. Применены равные интервалы для определения точных точек приближения функций механизма. Величина отклонения введена в функцию цели и рассматривается как неизвестный параметр при вычислений. Новые точные точки приближения функций определяются из уравнений, полученных путем дифференцирования функции цели. Процесс синтеза повторяется до тех пор пока абсолютная величина отклонения стабилизируется в заданных пределах отклонения. Результаты даны в таблицах и графиках.